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LIMIT ANALYSIS OF SPACE GRIDS

M. GRIGORIAN
Department of Structural Engineering, Arya-Mehr University of Technology, Tehran, Iran

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Abstract—The paper presents generalized, unique solutions for the limit design of pin-connected, double-layer space grids of regular construction, simply-supported along the sides of a rectangle and carrying a uniform concentration of normal nodal loading applied to the upper layer of the structure. The principal assumption upon which the foregoing results are obtained is that member forces increase monotonically with the applied loads, therefore insuring against the inducement of higher than ultimate loads in the compressive elements at anytime prior to structure failure. The virtual work method of the plastic analysis has been employed in conjunction with the techniques of the finite difference calculus to study the various collapse modes of the system. The work is concluded with an illustrative example to demonstrate the simplicity of the proposed solutions.

NOTATION

X, Y, Z	Standard mutually perpendicular coordinates (directions).
x, y	integers defining coordinates of a joint.
m, n	integers defining number of bays in X and Y directions respectively.
a, b	mesh lengths in X and Y directions respectively.
h	bay or truss height.
c	length of an inclined member.
ψ, ϕ, θ	direction cosines of an inclined member with respect to X, Y and Z axes respectively.
P	intensity of uniformly concentrated normal nodal load.
$P_{x,y}$	generalized normal joint load.
P_x, P_y	proportions of $P_{x,y}$ carried by trusses along X and Y directions respectively.
P_t, P_c, P_s	collapse load intensities in terms of tension, compression and web element strengths respectively.
$C, \bar{C}$	ultimate load capacities of upper chords in X and Y directions respectively.
$T, \bar{T}$	ultimate load capacities of lower chords in X and Y directions respectively.
S	ultimate load capacity of web members.
μ_c, μ_t	coefficients of plastic orthotropy for compression and tension chords respectively.
$C_{x,y}, \bar{C}_{x,y}$	upper chord forces in X and Y directions respectively.
$T_{x,y}, \bar{T}_{x,y}$	lower chord forces in X and Y directions respectively.
$Q_{x,y}, R_{x,y}, W_{x,y}, F_{x,y}$	inclined member forces meeting at any joint x, y.
$V_{x,y}$	vertical reaction along supports.
$Z_{x,y}$	virtual displacements in Z direction.
$\epsilon_{x,y}, \bar{\epsilon}_{x,y}$	virtual strains in X and Y directions respectively.
$\omega_{x,y}, \bar{\omega}_{x,y}$	virtual displacements in X and Y directions respectively.
$\gamma_{x,y}$	virtual twisting strain.
λ	maximum virtual displacement in Z direction.
E, E^{-1}	forward and backward shift operators respectively.
$\Delta = (E-1)$	forward difference operator.
$\nabla = (1 - E^{-1})$	backward difference operator.
$\boxed{\nabla} = \Delta - \nabla$	second central difference operator.

The remainder of the symbols are defined as they first appear in the paper.

INTRODUCTION

Space grids are 3-dimensional structural systems suited to cover large, column free areas while leaving free passage through their bays. The majority of space grids used in roof and floor construction are of the double-layer simply-supported type. In their simplest form, these structures consist of two geometrically identical, parallel, regular plane grids forming the top and bottom chord layers, separated by vertical and/or inclined shear posts placed between the two grids and ball-jointed to the horizontals at their points of intersection. In general there are two main types of double-layer grids: Trussed grids, composed of two sets of identical parallel trusses intersecting at constant angles, and, space grids, consisting of a combination of tetrahedra, octahedra or skeleton pyramids having rectangular or hexagonal bases. The general scheme of a rectangular base pyramid space grid together with its layout and a co-ordinates system is shown in Fig. 1.

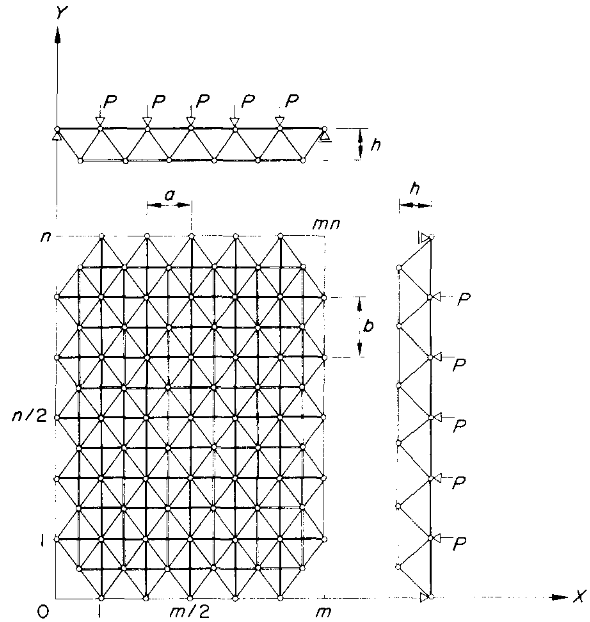

Fig. 1. Simply-supported regular rectangular space grid, layout and co-ordinates.

Because of their structural efficiency, aesthetic and economic potentials[1, 16] the use of double-layer grids as functional decking systems has become widespread in the past two decades. In spite of their popular use and the large number of technical papers dealing with the elastic analysis of space grids and similar structures, no conclusive effort seems to have been made to study the ultimate load behaviour of these structures under practical conditions, particularly with the intention of extending the applications of the plastic theory to the design of space grids with generalized properties in the two directions. Recently one such solution for a rectangular simply-supported trussed grid under uniform normal nodal loading was developed by the author[2], where use was made of the techniques of the finite difference calculus to formulate and solve the governing equilibrium equations of a Pratt type trussed grid. This solution, although of certain practical value is restricted in application to only trussed grids of the prescribed geometry, loading, and boundary conditions.

The pioneering effort in this field is due to Heki and Saka[3], who by resorting to an equivalent continuum try to transform their double layer space frame into an analoguous lattice plate under uniform transverse pressure. Their contribution provides only an approximate treatment of the particular case of isotropic space grids with continuous square boundaries. A similar but more practical account of the same problem is also given by Schmidt[4], where a two-stage analysis, based on a design by the strip method[5], is proposed with the conclusion that elastic designs not only require more material than plastic solutions, but in addition give rise to unstable load-deflexion characteristics past the ultimate load, (which is of course an undesirable mode of behaviour) and that coarse mesh equivalent trusses do not furnish the ultimate capacities of fine mesh truss systems. It appears therefore, that unless the mesh size is very small the continuum analogue would fail to provide reasonable answers for the internal forces of the constituent members of the actually discrete structural system.

The aim of this study is to present a simple, valid, generalized solution describing the ultimate load behaviour of simply-supported rectangular double layer space grids under continuous concentration of normal nodal forces applied over the entire surface of the structure. The study is begun by first investigating the collapse modes of the structure using the virtual work concept of plastic analysis and then verifying the validity of the proposed solutions from a lower-bound point of view. The coinciding upper and lower-bound results confirm the uniqueness of the calculated collapse loads. The approach adopted is that recently developed[6–10] to produce a number of generalized, unique, closed form solutions for the collapse load of torsionless plastically orthotropic grillages of regular formation with certain combinations of boundary support conditions along the sides of a parallelogram.

The simplicity of the proposed solutions may be seen to be due to the regular formation of the space grids which make the analysis conducive to the methods of discrete-field mechanics, as well as to the assumption that no premature failure due to connection dificiencies, brittleness and local buckling may occure at anytime before the plastic collapse of the structure.

UPPER-BOUND ANALYSIS

Assuming a sufficiently long yield plateau for the members of the space grid both in tension and compression, and precluding premature failure due to causes other than plastic yielding then two distinct modes of failure corresponding to the two basic types of members incorporated in the assembly of the structure may be postulated as in Fig. 2. The flexural type mode of collapse (Fig. 2a) corresponds to the extensions and contractions of the lower and upper chords respectively, and the shear type failure (Fig. 2b), corresponds to the stretching or shortening of the inclined web members. However, since the nature of the number of bays, m and n, whether odd or even, influences the flexural type collapse patterns thus affecting the numerical values of the corresponding failure loads, it becomes necessary to study eight combinations of such collapse mechanisms. In the foregoing study only the analysis of one such typical failure mode, i.e. that involving the plastic stretching of the lower chords with even number of bays in both directions is presented in detail. The analysis of the remaining cases being repetitious are not reported here, but their results are given without further elaboration.

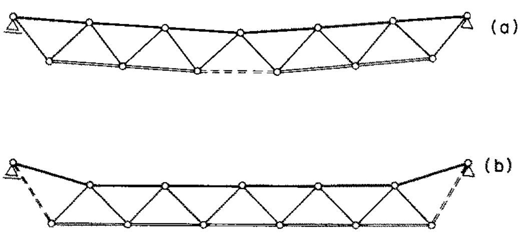

Fig. 2. Basic collapse modes; (a) flexural type failure, (b) shear type failure.

FLEXURAL TYPE COLLAPSE MODES

Referring to a doubly symmetric failure pattern caused by the tensile yielding of the lower chords located directly below central lines $x = m/2$ and $y = n/2$ it may be noted that each of the four collapsing segments being a rectangle will undergo a rigid body displacement with constant virtual twisting strain without stress defined by

$$\gamma_{x,y} = \Delta_x \Delta_y Z_{x,y}, \tag{1}$$

with corresponding direct virtual strains

$$\epsilon_{x,y} = (h/a^2)\Delta_x Z_{x,y}, \tag{2}$$

$$\bar{\epsilon}_{x,y} = (h/b^2)\Delta_y Z_{x,y}. \tag{3}$$

along the yielding chords of the lower network. Now considering the virtual displacements of the first quadrant of the collapsing grid bounded by the lines $x = 0$, $x = m/2$ and $y = 0$, $y = n/2$ and assigning a maximum virtual displacement λ to the central node of the structure at $x = m/2$, $y = n/2$ equation (1) would yield, upon integration and substitution, the virtual displacement surface,

$$Z_{x,y} = 4xy\lambda/mn \tag{4}$$

to which correspond the virtual extensions,

$$\omega_{x,y-1/2} = a\,.\,\epsilon_{x,y-1/2} = 4\lambda h(y-\tfrac{1}{2})/mna, \tag{5}$$

$$\bar{\omega}_{x-1/2,y} = b\,.\,\bar{\epsilon}_{x-1/2,y} = 4\lambda h(x-\tfrac{1}{2})/mnb, \tag{6}$$

Therefore, studying the postulated collapse mechanism by the virtual work method, it gives for a uniform concentration of normal nodal forces P at every joint xy,

$$\sum_{x=1}^{m/2-1}\sum_{y=1}^{n/2-1} P\,.\,Z_{x,y} + \frac{1}{2}\sum_{y=1}^{n/2-1} P\,.\,Z_{m/2,y} + \frac{1}{2}\sum_{x=1}^{m/2-1} P\,.\,Z_{x,n/2} + \frac{1}{4}P\,.\,Z_{m/2,n/2}$$
$$= \sum_{y=1}^{n/2} T\,.\,\omega_{x,y-1/2} + \sum_{x=1}^{m/2} \bar{T}\,.\,\bar{\omega}_{x-1/2,y}. \tag{7}$$

Working out the sums in equation (7), rearranging and simplifying eventually gives

$$P_t = 8h\left[\frac{1}{am^2} + \frac{\mu_t}{bn^2}\right]T, \tag{8}$$

as the failure load of the space grid in terms of the limit load-carrying capacities of the lower chord members. The same procedure may be repeated for grid systems with odd number of bays in the two directions to obtain the corresponding collapse load formula as:

$$P_t = 8h\left[\frac{1}{a(m^2-1)} + \frac{\mu_t}{b(n^2-1)}\right]T. \tag{9}$$

By introducing the auxiliary terms

$$\delta_m = [(-1)^m - 1]/2, \tag{10}$$

$$\delta_n = [(-1)^n - 1]/2, \tag{11}$$

solutions (8) and (9) may be combined to produce the following generalized solution for any combination of even and odd number of bays in the two directions:

$$P_t = 8h\left[\frac{1}{a(m^2 + \delta_m)} + \frac{\mu_t}{b(n^2 + \delta_n)}\right] T. \tag{12}$$

Formula (12) completes the mathematical solution for four combinations of collapse modes involving the plastic failure of the lower chords. The nature of the collapse modes concerning the compressive squashing of the upper chord members meeting at or crossing over the central lines $x = m/2$ and $y = n/2$ is identical to that described above, therefore performing a similar upper-bound analysis as for the preceding case, the limit load carrying capacity of the space grid in terms of the strengths of the compressive elements may be written as

$$P_c = 8h\left[\frac{1}{a(m^2 + \gamma_m + \delta_m)} + \frac{\mu_c}{b(n^2 + \gamma_n + \delta_n)}\right] C, \tag{13}$$

which with the aid of the auxiliary terms

$$\gamma_m = [(-1)^{m+1} - 1], \tag{14}$$

$$\gamma_n = [(-1)^{n+1} - 1], \tag{15}$$

combines the four upper-chord solutions into one. Obviously the better upper-bound collapse load of the structure up to this stage would be the smaller of the two values given by equations (12) and (13).

SHEAR TYPE COLLAPSE MODE

This type of failure may occur due to plastic deformations of the inclined web members of the boundary bays of the space grid. However, if for this type of collapse a symmetrical rigid body sinking of the structure bounded by the lines $x = 1$, $x = m - 1$, and $y = 1$, $y = n - 1$ is considered, then for a uniform virtual displacement $Z_{x,y} = \lambda$ of the collapsing segment, the following equation of virtual work may be written,

$$\sum_{x=1}^{m-1} \sum_{y=1}^{n-1} P \,.\, Z_{x,y} = 2 \sum_{y=1}^{n-1} 2S \,.\, \theta \,.\, Z_{1,y} + 2 \sum_{x=1}^{m-1} 2S \,.\, \theta \,.\, Z_{x,1} \tag{16}$$

giving

$$P_s = \frac{4h}{c}\left[\frac{1}{(m-1)} + \frac{1}{(n-1)}\right] S, \tag{17}$$

as the collapse load of the structure in terms of the limit load-carrying capacity of the inclined members.

By virtue of the work method equations (12), (13) and (17) constitute upper bounds to the collapse load of the structure, and are therefore unsafe, since the true collapse load may be less than or at best equal to those given by the three values P_t, P_c and P_s. However, to verify the validity of these solutions it is necessary to prove that the nature of the collapse loads as given by these equations is such that they give rise to statically admissible equilibrium fields at collapse, and that the resulting force fields satisfy the prescribed conditions of failure, which in the present case are the yield criteria:

$$T_{x,y} \leq T,\ \bar{T}_{x,y} \leq \mu_t T,\ C_{x,y} \leq C,\ \bar{C}_{x,y} \leq \mu_c C$$
$$F_{x,y} \leq S,\ W_{x,y} \leq S,\ Q_{x,y} \leq S,\ R_{x,y} \leq S. \tag{18}$$

To establish a safe distribution of forces leading to the final results (12), (13) and (17) the statical equilibrium of the space grid under consideration is studied as follows.

LOWER-BOUND ANALYSIS

In order to develop equations governing the statical equilibrium of the generalized module shown in Fig. 3, it is necessary to have expressions which relate the internal and external forces at the ends of a typical inclined member extending from any joint xy. Thus stating the conditions of statical equilibrium along the three mutually perpendicular directions X, Y and Z at joint x,y and $(x+\frac{1}{2})$, $(y+\frac{1}{2})$, it gives

$$Q_{x,y} + R_{x,y} + F_{x,y} + W_{x,y} - \frac{1}{\theta} P_{x,y} = 0, \tag{19}$$

$$Q_{x,y} + R_{x,y} - F_{x,y} - W_{x,y} + \frac{1}{\psi} \nabla_x C_{x,y} = 0, \tag{20}$$

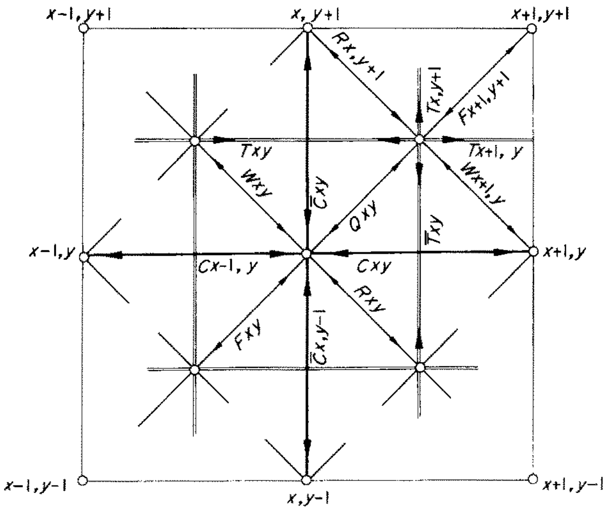

Fig. 3. Presentation of forces on a basic module.

$$Q_{x,y} - R_{x,y} - F_{x,y} + W_{x,y} + \frac{1}{\phi}\nabla_y \bar{C}_{x,y} = 0, \tag{21}$$

$$Q_{x,y} + E_y R_{x,y} + E_x E_y F_{x,y} + E_x W_{x,y} = 0, \tag{22}$$

$$Q_{x,y} + E_y R_{x,y} - E_x E_y F_{x,y} - E_x W_{x,y} + \frac{1}{\psi}\Delta_x T_{x,y} = 0, \tag{23}$$

$$Q_{x,y} - E_y R_{x,y} - E_x E_y F_{x,y} + E_x W_{x,y} + \frac{1}{\phi}\Delta_y \bar{T}_{x,y} = 0. \tag{24}$$

The pairs of equations (20) with (21) and (23) with (24) may be combined to read:

$$\frac{1}{\psi}\boxbslash_x C_{x,y} + \frac{1}{\phi}\boxbslash_y \bar{C}_{x,y} = (\Delta_x + \Delta_y)(F_{x,y} - Q_{x,y}) + (\Delta_x - \Delta_y)(W_{x,y} - R_{x,y}), \tag{25}$$

$$\frac{1}{\psi}\boxbslash_x T_{x,y} + \frac{1}{\phi}\boxbslash_y \bar{T}_{x,y} = (\Delta_x + \Delta_y)(E_x E_y F_{x,y} - Q_{x,y}) - (\nabla_x - \nabla_y)(E_y R_{x,y} - E_x W_{x,y}). \tag{26}$$

Now if the fully plastic axial strengths of the constituent members of the space grid are those indicated by the inequalities (18) it can be shown that the following distributions of forces not only satisfy the prescribed conditions of failure and the equations of equilibrium but also remain compatible with the boundary support conditions.

$$C_{x,y} = \frac{4C}{m^2 + \gamma_m + \delta_m}[(m-1)x - x^2 + (m-1)/2], \tag{27}$$

$$\bar{C}_{x,y} = \frac{4\bar{C}}{n^2 + \gamma_n + \delta_n}[(n-1)y - y^2 + (n-1)/2], \tag{28}$$

$$T_{x,y} = \frac{4T}{m^2 + \delta_m}[mx - x^2], \tag{29}$$

$$\bar{T}_{x,y} = \frac{4\bar{T}}{n^2 + \delta_n}[ny - y^2]. \tag{30}$$

Further, because of symmetry of shape and loading and the twistlessness of the basic module of the space grid, i.e. a rectangular base pyramid, the analysis may be reduced to statically determinate conditions following the assumptions of failure [11]. In other words the collapse load intensity $P_{x,y}$ may be considered to be composed of proportions P_x and P_y in the X and Y directions respectively, i.e.

$$P_{x,y} = P_x + P_y, \tag{31}$$

thus, implying that the shear force distribution in the inclined web members may be written as:

$$Q_{x,y} = \frac{1}{4\theta}[-P_x(m-1-2x) - P_y(n-1-2y)], \tag{32}$$

$$F_{x,y} = \frac{1}{4\theta}[P_x(m+1-2x) + P_y(n+1-2y)], \tag{33}$$

$$R_{x,y} = \frac{1}{4\theta}[-P_x(m-1-2x) + P_y(n+1-2y)], \tag{34}$$

$$W_{x,y} = \frac{1}{4\theta}[P_x(m+1-2x) - P_y(n-1-2y)]. \tag{35}$$

Substituting for the relevant force functions in equations (25) and (26) and performing the appropriate difference operations they give.

$$P_t = 8h\left[\frac{1}{a(m^2+\delta_m)} + \frac{\mu_t}{b(n^2+\delta_n)}\right]T, \tag{36}$$

$$P_c = 8h\left[\frac{1}{a(m^2+\gamma_m+\delta_m)} + \frac{\mu_c}{b(n^2+\gamma_n+\delta_n)}\right]C, \tag{37}$$

as the lower-bounds to the collapse load of the space grid in terms of tension and compression element strengths respectively, indicating that solutions (36) and (37) are unique, and that equations (27)–(35) describe the true distribution of member forces at collapse.

The usefulness of these lower-bound solutions is not restricted to obtaining unique collapse loads, but in addition they may be used to calculate the distribution of the reactions along the supports thus

$$V_{0,y} = +(W_{1,y} + F_{1,y})\theta = P_x(m-1)/2, \tag{38}$$

$$V_{m,y} = -(W_{m,y} + F_{m,y})\theta = P_x(m-1)/2, \tag{39}$$

$$V_{x,0} = +(F_{x,1} + R_{x,1})\theta = P_y(n-1)/2, \tag{40}$$

$$V_{x,n} = -(F_{x,n} + R_{x,n})\theta = P_y(n-1)/2. \tag{41}$$

The reactions along the supports are therefore uniform and their sum can be shown to be equal to the total load applied to the grid, i.e.

$$(n-1)(V_{0,y} + V_{m,y}) + (m-1)(V_{x,0} + V_{x,n}) = (m-1)(n-1)P. \tag{42}$$

Now by virtue of reaction equations (38)–(41)

$$S = \frac{1}{2\theta} V_{\max}, \tag{43}$$

which when substituted in (42) gives,

$$P = \frac{4h}{c}\left[\frac{1}{(m-1)} + \frac{1}{(n-1)}\right]S, \tag{44}$$

which confirms equation (17) subject to condition

$$\frac{P_x}{P_y} = \frac{n-1}{m-1}.$$

EXAMPLE

Consider the collapse load of a square ($m = n = 10$) isotropic ($\mu_c = \mu_t = 1$) uniform space grid ($a = b = c$), then by the application of the proposed unique solutions

$$h = a/\sqrt{2}, \quad \gamma_m = -2, \qquad \delta_m = 0$$

$$P_c = \frac{8\sqrt{2}}{100}T, \quad P_c = \frac{8\sqrt{2}}{98}C, \quad P_s = \frac{8\sqrt{2}}{198}S,$$

which are associated with the statically admissible distributions,

$$C_{x,y} = \frac{P}{2\sqrt{2}}(9x - x^2 - 9/2), \quad T_{x,y} = \frac{P}{2\sqrt{2}}(9x - x^2),$$

$$\bar{C}_{x,y} = \frac{P}{2\sqrt{2}}(9y - y^2 - 9/2), \quad \bar{T}_{x,y} = \frac{P}{2\sqrt{2}}(9y - y^2),$$

$$Q_{x,y} = -\frac{P}{\sqrt{2}}(9 - x - y), \qquad F_{x,y} = +\frac{P}{\sqrt{2}}(11 - x - y),$$

$$R_{x,y} = -\frac{P}{\sqrt{2}}(-x + y), \qquad W_{x,y} = +\frac{P}{\sqrt{2}}(-x + y),$$

with constant reaction distribution along the four sides:

$$V_{0,y} = V_{m,y} = V_{x,0} = V_{x,n} = 9P/4.$$

CONCLUDING REMARKS

The idea of applying the proposed plastic method of analysis to the design of space grids sounds attractive when its analytic simplicity and inherent advantages are considered. Plastic theory has already been used successfully for the design of continuous trussed girders and indeterminate trussed frames [12]. The concept becomes feasible realizing that the various members of the space grid can be selected in accordance with the rules especially worked out for the purpose [12–13] for instance in the case of compression elements, the member characteristics may be controlled against either the Melbourne regression formula [13–14]

$$P_C/P_S = 0{\cdot}90\, P_R/P_Y + 0{\cdot}162, \tag{45}$$

or Merchant's [15] semi-empirical equation

$$P_C/P_S = 0{\cdot}82\, P_R/P_Y + 0{\cdot}172, \tag{46}$$

where in both equations P_R is the Rankine load given by

$$1/P_R = 1/P_Y + 1/P_E$$

in which P_Y is the squash or yield load, P_E the elastic critical load and $P_c = C$, $\bar{C}$ or web force, the collapse load or maximum axial compression.

In comparing the collapse load data presented in this work, it may be seen that for any given space grid of the type described above, the failure load would be given by the smaller of the three values P_c, P_t and P_s. In considering the design problem, these load carrying capacities may be chosen in such a way as to produce identical collapse loads for all possible failure modes. Ideally, if this is done, further economy may be achieved by sizing the members in accordance with the corresponding distribution functions. Although the present work furnishes sufficient information for the selection of the basic members, the plastic design of the system under consideration may be approached in several practical ways. For instance, taking into account the restraining effects of the roof deck, the load-carrying capacity of the upper layer may be increased considerably. Indeed by proper selection of decking, e.g. reinforced concrete roofing acting monolithically with the top chords, the problem of premature buckling of the horizontal compression members may entirely be discarded. The uniform distribution of the reactions along the supports also indicate a more economical loading system for the design of the edge beams which may in turn be designed using the plastic methods of structural analysis[9].

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